DEVELOPMENT OF NOVEL STOCHASTIC MESHLESS METHODS FOR LINEAR AND NONLINEAR PROBLEMS IN STRUCTURAL MECHANICS

A THESIS

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Abstract

The current study addresses the need for developing a better numerical method for stochastic analysis using meshless method. The primary objective of the study is to develop computationally efficient and accurate stochastic meshless method to capture the probabilistic moments and distributions of the structural responses. The meshless method used in the current work is the element-free Galerkin method (EFGM). The present study addresses a wide range of problems in structural mechanics including linear elastic boundary value problems, linear eigenvalue problems, and geometric nonlinear problems with von-K$\grave{a}$rm$\grave{a}$n strains, by developing suitable techniques for each. This work considers randomness associated with Young's modulus and force, and models them as homogeneous random fields. The study deals with normal, truncated normal, or lognormal distribution characteristics for the input random field. This study assumes an exponentially decaying auto-correlation kernel for the random fields considered. Cross-correlation between the random variables is not accounted in this study. The current work uses moving least squares-based shape function method to discretize the input random field.

Initially, the study considers 1D and 2D boundary value problems in linear elastic structural mechanics. For such problems, the study develops an improved response function (IRF) based stochastic element free Galerkin method (SEFGM). This method has its roots in existing ad-hoc response function (ARF) method, which is proposed in stochastic finite element method. The IRF method assumes the total response as the sum of deterministic and stochastic parts. The stochastic part is named the IRF and is a function of the complete set of discretized random variables. Taylor series expansion is used for approximating the stiffness matrix and force vector. The expressions for the deterministic part and IRF are derived from stochastic discretized set of equations. The probabilistic moments and distributions of response are evaluated by performing simulations on the derived expression of response. A few numerical examples are solved, and the results are compared with those of Monte Carlo simulation (MCS) and perturbation techniques. A comparison is also made with the ARF method proposed in SEFGM framework. The results are found to be matching with the MCS results for the range of coefficient of variation (CV) values, regardless of the correlation length parameters of the random fields. The statistical characteristics of

the response, obtained from IRF method is found to be superior to those obtained from perturbation and the proposed ARF based SEFGM. From a study on computational times normalized with the time required for MCS, it is found that the proposed IRF method requires lesser time compared to ARF and MCS methods. The perturbation method needs the least time. However, it fails to produce the probabilistic distributions of the response.

As the second objective of the study, a method for stochastic eigenvalue problems in structural mechanics, is developed. Use of the IRF method in such problems needs solving two characteristic equations. The first equation can be used to find the deterministic solution, whereas the second equation becomes unsolvable since it needs inversion of a matrix, containing stiffness derivatives alone, which is singular. Hence, the current study proposes a method which uses Taylor series expansion for the stiffness approximation. The method does not use any approximation for stochastic modelling of response. The method evaluates the probabilistic moments and distributions of eigenvalues from the samples obtained by performing simulations on the stochastic set of characteristic equations in which Taylor series approximation of stiffness matrix is substituted. A few eigenvalue problems are solved using the proposed method, and the results are compared with the MCS and perturbation based SEFGM. It is observed that the proposed method gives comparable results with those of MCS and perturbation techniques for the range of *CV* values considered, irrespective of the correlation length parameters of the input random field. Computational efficiency of the proposed method is also established from a study of normalized computational times.

As the third objective of the study, a high-dimensional model representation (HDMR)-based stochastic meshless method for geometric nonlinear analysis of structures with von-K*à*rm*à*n strains is developed. The IRF method developed for linear elastic problems and the method proposed for stochastic eigenvalue problems cannot be used for nonlinear problems since those methods lead to convergence issues. The study also proposes a perturbation method for geometric nonlinear problems for evaluating the first and second probabilistic moments of response alone. HDMR decomposes the displacement response as a series of individual component functions. Evaluation of each component function needs only a few deterministic analyses to be performed at a set of sample points. Simulations performed on the derived expression of response can be used to calculate its probabilistic characteristics. This study investigates the use of first order and second order HDMR based SEFGM; it also compares the use of both additive and factorized HDMR techniques. In the two numerical examples solved, the probabilistic moments of deflection are evaluated using HDMR, and are found to be matching with MCS values for the range of *CV* considered, at different correlation length parameters. From a study of normalized computational times, the computational efficiency of HDMR techniques is also established.