Breather modes in spin chains: A study on the geometry and dynamics of certain special magnon modes

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Rahul O. R.

Department of Physics

Indian Institute of Space Science and Technology

Thiruvananthapuram, India

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Abstract

Geometry is inherent and well rooted in various disciplines of science. Integrable nonlinear system is one among them. They are exactly solvable with soliton solutions, and are naturally associated with differential curves and surfaces. One of the remarkable feature of integrable system is the *recurrence* property, wherein the system returns to its initial state through a rather complicated nonlinear evolution. This was observed for the first time as a paradox in the celebrated Fermi-Pasta-Ulam-Tsingou (FPUT) experiment. The paradox eventually led to the discovery of solitons and laid the foundation for the entire subject of integrability. The recurrence phenomena can be modelled by spatially periodic *breather* solution to the nonlinear Schrödinger equation (NLSE), more precisely, Akhmediev breather, wherein the system recovers the initial state in its time evolution. Being the governing model for a variety of physical systems, NLSE is well studied with a substantial amount of literature.

In this thesis we examine geometrical aspects of the NLSE in the context of breather solutions. The following nonlinear systems are studied owing to their close relationship with NLSE:

- ∘ Classical 1-d Heisenberg Ferromagnet (HF),
- ∘ Vortex filament in fluid under localized induction approximation (LIA).

In classical HF model we examine the breather excitation in detail. An explicit expression for the spin breather is presented. Spatially periodic case — a counter part of 'Akhmediev breather', is studied in particular. This special *magnon* mode leads to a *recurrence* phenomena in the HF model. In the background spin field the spin vectors take 'two' complete turns along the chain between its ends. During the breather excitation the spin chain continuously transforms to a configuration wherein the net turn becomes 'zero'. This peculiar geometrical feature is interpreted as a manifestation of the 'belt trick', which demonstrates the triviality of 4π rotation. Magnon mode is visualized in $\mathrm{SU}(2)$ group manifold along with a description of topological sectors and their energy lower bounds. In this breather mode the background spin configuration is a static field. Moreover, the initial and final configuration are exactly identical in the context of recurrence process.

Further, we present an explicit expression for a similar spin breather for which the background spin configuration is a dynamical field. This magnon mode is qualitatively different

in the sense that the recurrence process introduces an additional global rotation in the spin chain. That means the recurrence is not exact as in the previous case. This measurable change is treated as the 'trace' being left in the system during a breather excitation.

Mathematical framework of soliton theory associates each soliton solution with a surface in Euclidean 3-space. Geodesic on this surface is a space curve in $\mathbb{R}^3$ thereby providing a geometrical picture of the soliton solution. From a physics point of view, such a moving space curve approximately describes the dynamics of a vortex filament in fluid under LIA scheme. We obtain explicitly the breather excitation over a helical curve. Corresponding complex field of NLSE is also examined. We show that this is in fact a new *breather* solution to the NLSE. Specifically, the associated space curve is shown to have periodic *knot* formation in its time evolution. Previously known knotted solutions are of invariant shape which are associated with periodic solutions to the NLSE — more precisely, Kida class of solutions. We emphasize that a knot structure associated with a breather solution has so far not been witnessed. Spatially periodic case of this breather solution turn out to be a Galilean transformed version of the 'Akhmediev breather'. Their background space curves are a helix and a circle respectively. A circle and a helix are associated with plane wave solutions that are related through a Galilean transformation. We show that spatially periodic breathers of the Akhmediev type can also be obtained from a wave solution associated with a circle, through a Galilean transformation followed by a Darboux transformation. The general question of permutability of the two transforms, however remains open.